**#1** (10 Points)

**Is the following function a proper distance function? Why? Explain your answer. Measure the distance between (0, 0, 0), (0, 1, 0), (0, 1, 1), and (1, 1, 1)**

**Solution:**

**X= (0,0,0), Y= (0,1,0), Z= (0,1,1) and A= (1,1,1)**

**For any distance function to work the following conditions must be satisfied:**

1. Non-negativity or separation axiom



1. Identity of indiscernible
2. Symmetry
3. Subadditivity or triangle inequality

|  |  |
| --- | --- |
| Using given distance function, |  |
| The distance between X (0,0,0) & Y (0,1,0) => d (x, y) | = (|0 – 0|2 + |0 – 1|2 + |0 - 0|2 )  = (1)2  = 1 |
| The distance between Y (0,1,0) & X (0,0,0) => d (y, x) | = (|0 – 0|2 + |1 – 0|2 + |0 – 0|2)  = (1)2  = 1 |
|  |  |
| The distance between Y (0,1,0) & Z (0,1,1) => d (y, z) | = (|0 – 0|2 + |1 – 1|2 + |0 – 1|2)  = (1)2  = 1 |
| The distance between Z (0,1,1) & Y (0,1,0) => d (z, y) | = (|0 – 0|2 + |1 – 1|2 + |1 - 0|2) |

= (1)2

= 1

The distance between Z (0,1,1) & X (0,0,0) => d (z, x) = (|0 – 0|2 +|1 – 0|2 + |1 – 0|2)32

= 1 + 1

= 2

The distance between X (0,0,0) & Z (0,1,1) => d (x, z) = (|0 – 0|2 +|0 – 1|2 + |0 – 1|2)

= 1 + 1

= 2

The distance between X(0,0,0) & A(1,1,1) = (|0 – 1|2 + (|0 – 1|2  + (|0 – 1|2 )

= 1 + 1 + 1

= 3

The distance between A(1,1,1) & X(0,0,0) = (|1 – 0|2 + (|1 – 0|2  + (|1 – 0|2 )

= 1 + 1 + 1

= 3

The distance between A(1,1,1) & Y(0,1,0) = (|1 - 0|2 + |1 - 1|2 + |1 - 0|2)

= 1 + 0 + 1

= 2

The distance between Y(0,1,0) & A(1,1,1) = (|0 - 1|2 + |1 - 1|2 + |0 - 1|2)

= 1 + 0 + 1

= 2

The distance between A(1,1,1) & Z(0,1,1) = (|1 - 0|2 + |1 - 1|2 + |1 - 1|2)

= 1 + 0 + 0

= 1

The distance between Z(0,1,1) & A(1,1,1) = (|0 - 1|2 + |1 - 1|2 + |1 - 1|2)

= 1 + 0 + 0

= 1

Checking validity of the distance function properties on the distance values calculated using given distance function.

1. d (x, y) ≥ 0, d (y, x) ≥ 0, d (y, z) ≥ 0, d (z, y) ≥ 0, d (z, x) ≥ 0, d (x, z) ≥ 0.

Clearly are satisfied.

and



1. d (x, y) = d (y, x), d (y, z) = d (z, y), d (z, x) = d (x, z)
2. Clearly Picture 722 is satisfied.
3. d(x,y) = 1 , d(y,z) = 1 , d(z,x) = 2

d(z,x) ≤ d(x,y) + d(y,z)

=> 2 ≤ (1 + 1) -> is clearly satisfied

d(a,x) = 3, d(a,z) = 1, d(z,x) = 2

d(z,x) ≤ d(a,x) + d(a,z)

=> 2 ≤ 3 + 1 -> is clearly violated

d(a,y) = 2, d(a,x) = 3 , d(x,y) = 1

d(a,y) ≤ d(a,x) + d(x,y)

=> 2 ≤ 3 + 1 -> is clearly satisfied

d(a,y) = 2, d(a,z) = 1 , d(z,y) = 1

d(a,y) ≤ d(a,z) + d(z,y)

=> 2 ≤ (1 + 1) -> is clearly satisfied

As per above calculations and observations, given distance function satisfies all the 4 conditions. Therefore, given function **is a proper distance function.**

**# 8**(10 Points)

**The following table shows the population and the actual current prevalence rate of COVID19 in the US, Italy and Spain.**

**Considering only the three countries (US, Italy and Spain) use the table to answer the following questions:**

1. **Estimate the number of cases in the US, Italy and Spain.**
2. **Given that a person is living in the US, what is the probability that the person is infected with COVID19.**
3. **Given that a person is diagnosed with the COVID19, what is the probability that the person lives in the US.**

|  |  |  |
| --- | --- | --- |
|  | **Population** | **Prevalence** |
|  | rounded to nearest | Cases |
|  | **Million** | **Per Million** |
| **US** | 331 | 381.24 |
| **Italy** | 60 | 1463.97 |
| **Spain** | 47 | 1590.24 |

**Solution:**

**A:** Estimated number of cases in **US** = 381.24 \* 331 = **126190.44**

Estimated number of cases in **Italy** = 1463.97 \* 60 = **87838.2**

Estimated number of cases in **Spain** = 1590.24 \* 47 = **74741.28**

**B:** The probability of a person being infected with COVID-19 given that he lives in the US =

No of infected people / Total no of people = **126190.44 / 331000000 = 0.00038124**

**C:** Probability of a person lives in the US given he / she is diagnosed with the COVID19 =

P(US| COVID-19) = (P(COVID-19 |US) \* P(US))/ P(COVID-19)

P(COVID-19) = total no of cases in US, Italy & Spain / total population of US, Italy & Spain

= (126190.44 + 87838.2 + 74741.28) / ((331 + 60 + 47) \* 106)

= 288769.92 / 438000000

= **0.00065929205**

P(COVID-19 |US) = total no of people infected in US / Total population of US

= 126190.44 / 331000000

**= 0.00038124**

P(US) = population of US / total population of US, Italy & Spain

= 331 \* 106 / ((331 + 60 + 47) \* 106)

= 331 / 438

= **0.75570776255**

Upon substituting the values in the equation, we get

P(US| COVID-19) = (0.00038124 \* 0.75570776255) / 0.00065929205

= **0.43699302516**

**Submitted By:**

**Name:** Sanam Sritam Jena

**CWID:** 10454295